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ABSTRACT

In applications of phased array antennas for limited scan, sub-array schemes prove to be useful in reducing costs and space.^{1,2} An investigation of two such techniques, namely the space fed lens and a constrained feed system were conducted.

Radiation patterns for a flat space-fed lens have been obtained and it has been shown¹ that this system compares favorably in performance with the spherical lens, the latter being more difficult and expensive to construct in a space environment. Results on the effects of surface perturbations in the lens and on techniques for phase correcting at the feed for these perturbations are presented.

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Technical Information Officer

I. INTRODUCTION

Many military and civilian applications of phased array antennas require less than full sector scan. Two general classes of sub-array systems used for this purpose are²: (1) an array near the focal point of a lens or reflector and (2) a constrained feed system which suppresses grating lobes.

The first part of this investigation will deal with one-dimensional lens systems. Computer programs have been written to give results on a spherical lens system (this system was previously studied by Borgiotti³) and for a flat lens system¹. Antenna radiation patterns have been obtained for both systems as a function of scan angle³. The effects of mechanical deformation of the flat lens surface are presented along with results on one possible approach to correcting for these deformations by the feed system. The technique involves introducing phase corrections at each of the sub-array feed points. Results are presented on the improvement in main lobe amplitude, beamwidth and first side lobe level as a function of the amplitude of these deformations. The form of the deformation is assumed to be sinusoidal with distance along the array and results are given also as a functions of the "frequency" of this sinusoid.

II. RESULTS ON THE ONE DIMENSIONAL LENS

Figure 1 shows the lens system analyzed by Borgiotti³. A Butler Matrix is used to form multiple beams which are directed at the lens. The source is "phase-focused" to the center of the lens which, in turn, is focused at the center of the source. Computer programs were written to give results on this system as well as the flat lens system¹.

One of the possible difficulties with the space radar array involves mechanical deformations in the flat lens surface. To investigate this effect the computer programs were modified so that either sine or cosine surface deformations of arbitrary amplitude could be investigated. Figures (2) and (3) show radiation patterns for peak-to-peak deformations (cosine) of 0.1λ with a period of twice the lens length and 20 times the lens length, respectively. Similar results for peak-to-peak deformation of λ are shown in Figures (4) and (5). However, in this latter case a mechanical period of 20 results in an unusable pattern. Results are thus shown for a period of 2 in Figure (5).

One of the possible advantages of the space fed lens concept involves the possible correction for such deformations at the feed. This concept was investigated in only a preliminary way. Assume that the sub-array functions, F_n , at the output of the lens are known. If surface deformations, y_n , at each element are assumed perpendicular to the lens face then the radiation pattern is

$$P(u) = \sum_n^1 p_n e^{jn \frac{u}{M}} + j k y_n \cos \theta$$

where

$$u = k d \sin \theta$$

$$D = M d$$

$d \sim$ element spacing

the case $u_0 = 0$ with orthogonal f_n is particularly simple

since

$$\sum_p^1 f_p f_p^* - M(q - \ell) = \partial_{q\ell}$$

which leads to

$$\alpha_\ell = -k Y_\ell \cos \theta_0$$

which is intuitively satisfying and somewhat obvious.

Figures (6) - (11) show results of applying this technique to a one dimensional flat-lens system. This system has 107 radiating elements in the lens and is fed by a focused array driven by a Butler Matrix. Figure (6) gives results on the amplitude of the main beam (in DB) of the radiation pattern as a function of the amplitude of the mechanical deformation (normalized to wavelength) for unity mechanical "frequency". This corresponds to one cycle over the length of the lens. The lower and upper points show the main beam amplitude without and with the appropriate phase corrections at the sub-array feed points. Figure (7) gives results for the amplitude (in DB) of the first side-lobe. The width of the main beam as determined by the 3 DB points was also investigated. There was little degradation and hence little correction was possible in this quantity therefore Figure (6) is, to a reasonable approximation, an indication of the gain improvement by this phase correction scheme.

Figures (8) and (9) show similar results for a mechanical "frequency" of 5. The degradation in array performance both with and without phase correction at the source is greater. Figures (10) and (11) show results for a mechanical "frequency" of 10 and in this case the amount of degradation is substantial and virtually no correction is possible. These results indicate that as the mechanical "frequency" approaches or exceeds the number of sub-

Using notations similar to Dufort⁴ we define

$F_n \sim$ output to each antenna from a given sub-array input with all sub-array driven and with no surface deformations.

$$F_{no} = \sum_{\ell=0}^M f_n - M \ell z_{\ell o}$$

where

$$z_{\ell o} = |z_{\ell o}| e^{-j\alpha_\ell u_o}$$

thus

$$P_o(u) = \sum_n \sum_{\ell=0}^M f_n - M \ell z_{\ell o} e^{j \frac{n}{M} u} \\ \sum_{\ell} |z_{\ell o}| e^{j \alpha_\ell (u - u_o)} \sum_p F_p e^{jp \frac{u}{M}}$$

where the second factor is clearly the individual sub-array pattern. If with surface deformations we let

$$z_\ell = z_{\ell o} e^{j\alpha_\ell}$$

So that α_ℓ can be set to compensate for these deformations then a similar development yields

$$P(u) = \sum_n \sum_{\ell=0}^M f_n - n \alpha_\ell e^{j(\frac{n}{M} - \ell) u} e^{j\alpha_\ell (u - u_o)} |z_{\ell o}| e^{j\alpha_\ell} e^{jk y_n \cos \theta} \\ = \sum_p^M f_p e^{jp \frac{u}{M}} \sum_{\ell=0}^M |z_{\ell o}| e^{j\alpha_\ell} e^{j\alpha_\ell (u - u_o)} e^{jk y_p + M \ell \cos \theta}$$

If, we expand the last term as follows:

$$e^{jk y_n \cos \theta} = \sum_q^M a_q f_{n-Mq}^*$$

where u_o is the direction of the main beam

$$P(u_o) = \sum_q^M |z_{\ell o}| e^{j\alpha_\ell} \sum_q^M a_q \sum_p^M f_p^* f_{p-M(q-\ell)} e^{j \frac{p}{M} u_o}$$

and we wish to choose α_ℓ so that

$$P(u_o) = P_o(u_o)$$

III. RESULTS ON THE TWO DIMENSIONAL LENS

Computer programs were developed to calculate the radiation pattern of the two dimensional lens with parameters equivalent to the one dimensional lens of the previous section. Figures (12) - (15) show radiation patterns for this case with no mechanical deformation of the lens surface. These patterns are obtained along the X-axis (0°), along a 45° line to the X and Y axis, along the Y-axis (90°) and at 120° to the x-axis. The patterns along the X and Y axis corresponding to the one-dimensional lens case. Figures (16-19) show similar results for the case when only one of the sub-arrays is driven and again these results agree with the one dimensional case.

Figures (20) - (23) show patterns with mechanical deformation in both the X and Y directions of amplitude = 1 (normalized to wavelength) and of mechanical "frequency" = 2. The pattern along the X and Y axis are as expected but the pattern at 45° shows a narrow side-lobe which is only about 10 DB below the main beam. This is a significant problem. Figures (24) - (27) show results for the same deformation amplitude but at a mechanical frequency = 8. The pattern at 45° again shows a significant side-lobe, also our preliminary investigations seem to indicated that applying the phase correction scheme at the feed does little to diminish this sidelobe and; yet, the patterns along the X and Y directions show improvement consistent with the one-dimensional case. This problem needs further study.

arrays the performance of the array degrades rapidly with the amplitude of the mechanical deformations and more importantly, the technique of phase correction at the feed is virtually useless. Clearly this can be used to define the necessary mechanical properties of the lens structure.

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1. Caron, Paul R., "Overlapped Sub-array Techniques for Use in a Space Radar" Final Report, USAF-ASEE Summer Faculty Research Program, 1978.
2. Mailloux, R. J., Trends in Array Antenna Research, RADC-TR-77-195, June 1977.
3. Borgiotti, G. V., (1977), An Antenna for Limited Scan in One Plane: Design Criteria and Numerical Simulation, IEEE Trans. AP-25, (No. 2), 232-243.
4. Dufort, E. C. (1978), Constrained Feeds for Limited Scan Arrays, IEEE Trans. AP-26, (No. 3), 407-413.

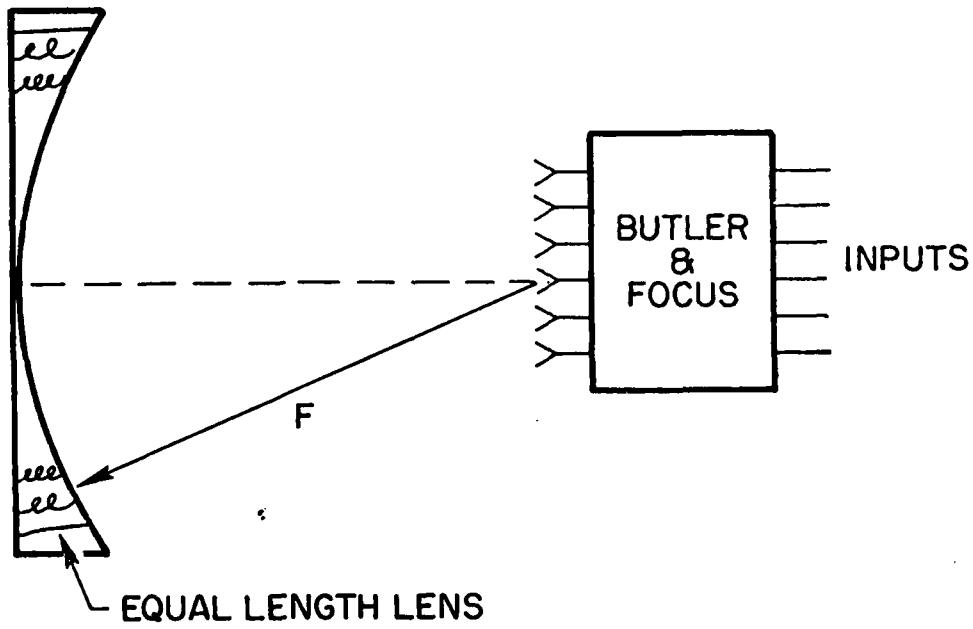


FIG. 1 - THE SPHERICAL LENS CONFIGURATION

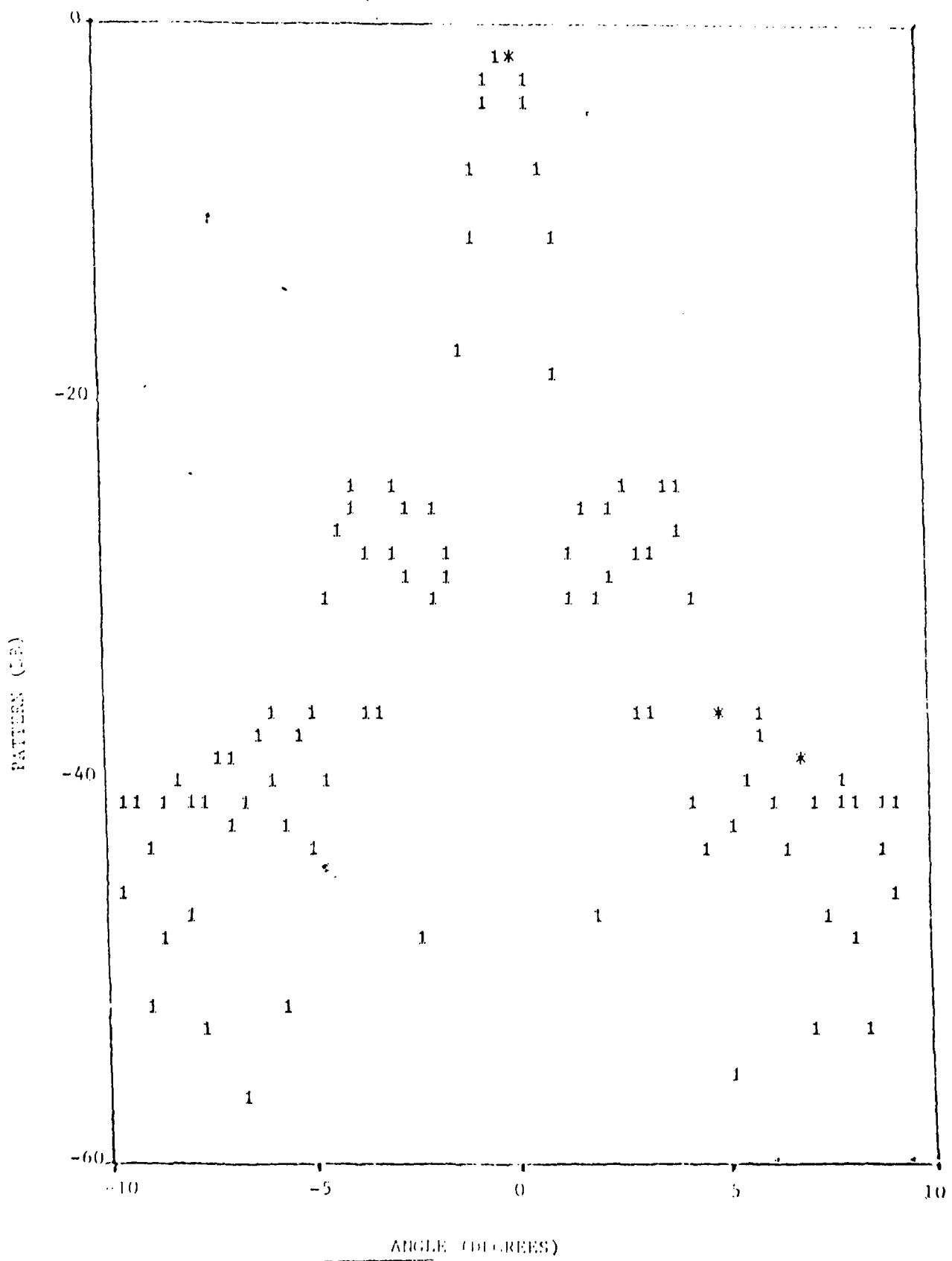


FIG. 2 - RADIATION PATTERN OF FLAT LENS WITH PROGRESSIVE PHASE = 0° AND WITH PEAK-TO-PEAK MECHANICAL DEFORMATION OF 0.1λ WITH PERIOD OF TWICE ARRAY LENGTH (COSTINE DEFORMATION).

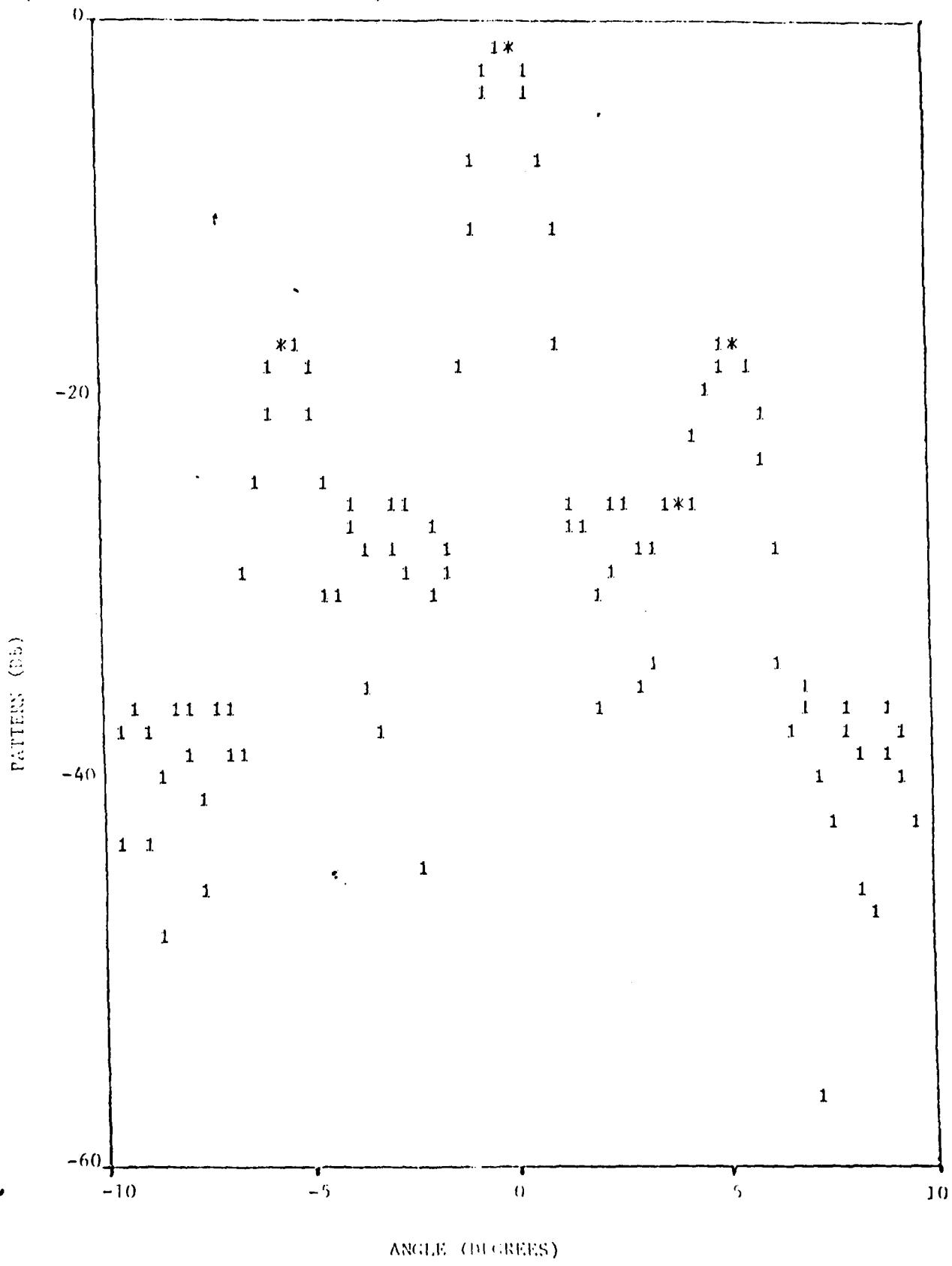


FIG. 3 - RADIATION PATTERN OF FLAT LENS WITH PROGRESSIVE PHASE = 0° AND WITH PEAK-TO-PEAK MECHANICAL DEFORMATION OF 0.1λ WITH PERIOD OF 20 TIMES ARRAY LENGTH (COSTIVE DEFORMATION).

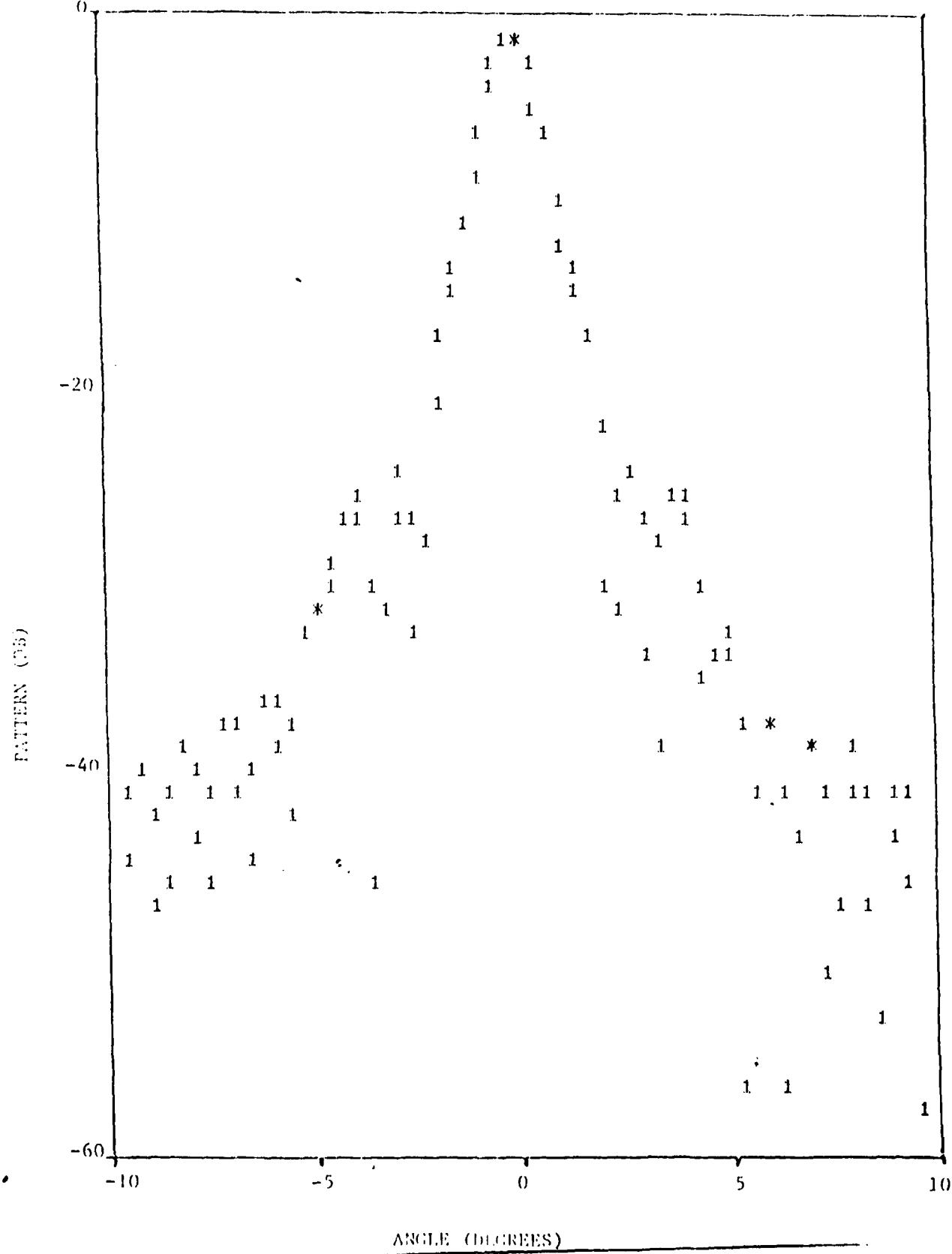


FIG. 4 - RADIATION PATTERN OF FLAT LENS WITH PROGRESSIVE PHASE = 0° AND WITH PEAK-TO-PEAK MECHANICAL DEFORMATION OF 0.5λ WITH PERIOD OF TWICE ARRAY LENGTH (COSINE DEFORMATION).

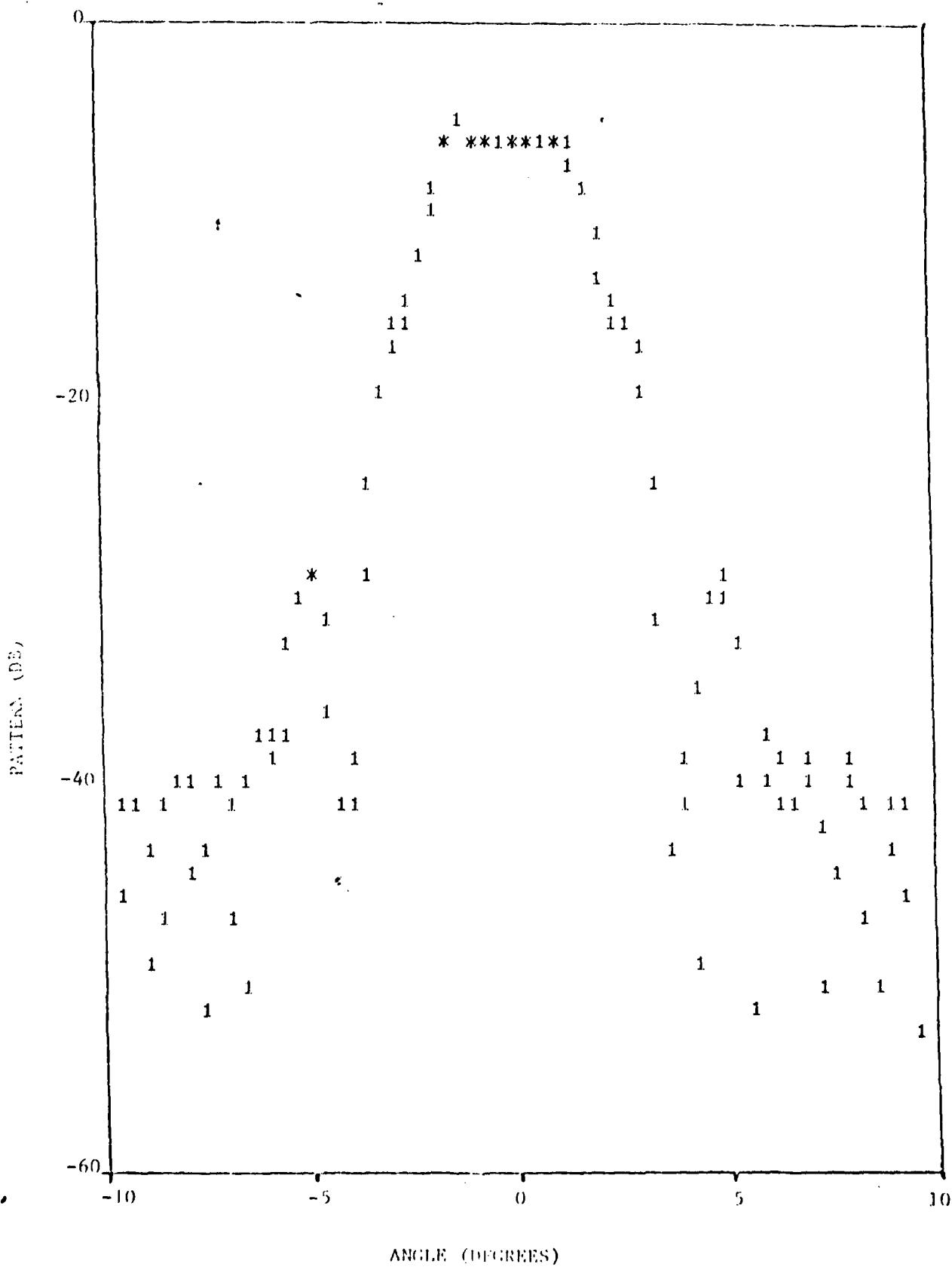


FIG. 5 - RADIATION PATTERN OF FLAT LENS WITH PROGRESSIVE PHASE = 0° AND WITH PEAK-TO-PEAK MECHANICAL DEFORMATION OF 0.5λ WITH PERIOD OF 4 TIMES ARRAY LENGTH (COSINE DEFORMATION).

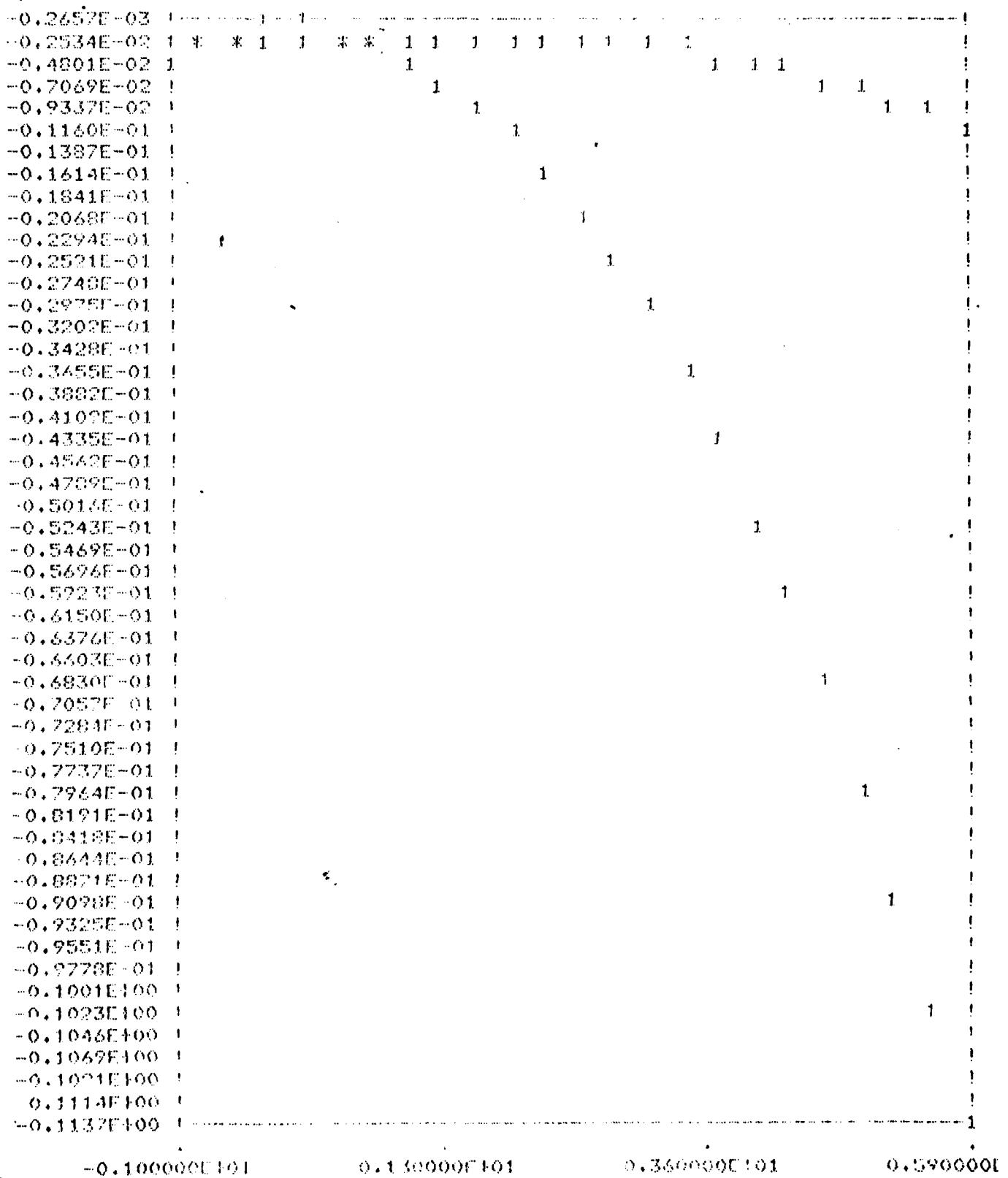


FIG. 6 - MAIN BEAM (DB), WITHOUT (UPPER) AND WITH (LOWER) PHASE CORRECTION AT SUB-ARRAY FEED, AS A FUNCTION OF AMPLITUDE OF MECHANICAL DEFORMATION FOR MECHANICAL "FREQUENCY" = 1.

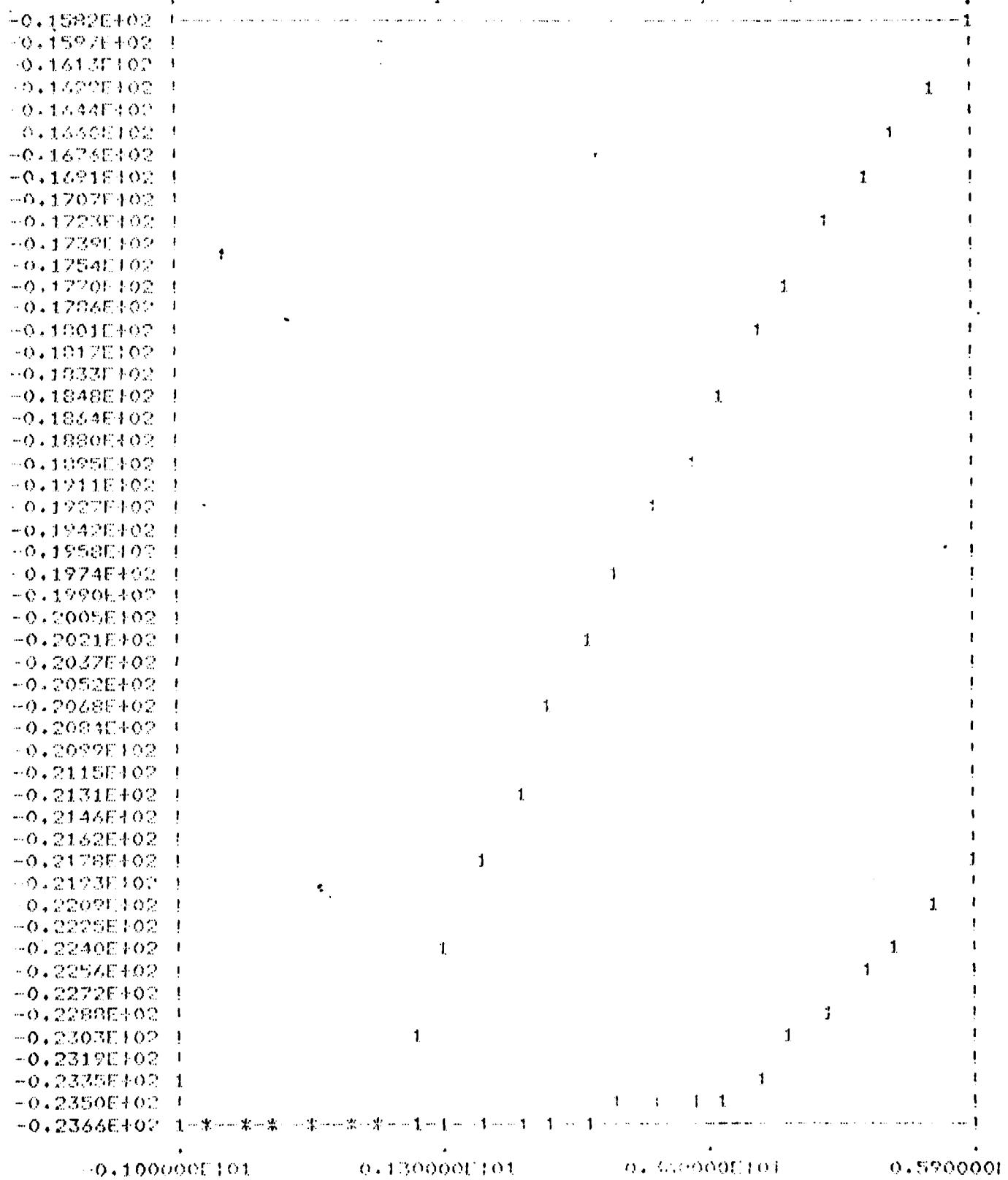


FIG. 7 - FIRST SIDE LOBE (DB), WITHOUT (UPPER) AND WITH (LOWER) PHASE CORRECTION AT SUB-ARRAY FEED, AS A FUNCTION OF AMPLITUDE OF MECHANICAL DEFORMATION FOR MECHANICAL "FREQUENCY" = 1.

$-0.2614E-03$	$-0.1140E-01$	$* * *$	1	$* * *$	1	1	1	1
$-0.2270E-01$	1			1	1	1	1	1
$-0.3391E-01$	1				1		1	1
$-0.4513E-01$	1					1		
$-0.5635E-01$	1						1	
$-0.6756E-01$	1					1		
$-0.7878E-01$	1						1	
$-0.9000E-01$	1					1		
$-0.1012E+00$	1						1	
$-0.1124E+00$	1							1
$-0.1236E+00$	1							
$-0.1348E+00$	1							
$-0.1461E+00$	1							
$-0.1573E+00$	1							
$-0.1685E+00$	1							
$-0.1797E+00$	1							
$-0.1909E+00$	1							
$-0.2022E+00$	1							
$-0.2134E+00$	1							
$-0.2246E+00$	1							
$-0.2358E+00$	1							
$-0.2470E+00$	1							
$-0.2583E+00$	1							
$-0.2695E+00$	1							
$-0.2807E+00$	1							
$-0.2919E+00$	1							
$-0.3031E+00$	1							
$-0.3143E+00$	1							
$-0.3256E+00$	1							
$-0.3368E+00$	1							
$-0.3480E+00$	1							
$-0.3592E+00$	1							
$-0.3704E+00$	1							
$-0.3816E+00$	1							
$-0.3928E+00$	1							
$-0.4041E+00$	1							
$-0.4153E+00$	1							
$-0.4265E+00$	1							
$-0.4377E+00$	1							
$-0.4489E+00$	1							
$-0.4601E+00$	1							
$-0.4714E+00$	1							
$-0.4826E+00$	1							
$-0.4938E+00$	1							
$-0.5050E+00$	1							
$-0.5162E+00$	1							
$-0.5275E+00$	1							
$-0.5387E+00$	1							
$-0.5499E+00$	1							
$-0.5611E+00$	1							
$-0.100000E+01$	$0.130000E+01$	$0.340000E+01$	$0.590000E+01$					

FIG. 8 - MAIN BEAM (DB), WITHOUT (UPPER) AND WITH (LOWER) PHASE CORRECTION AT SUB-ARRAY FEED, AS A FUNCTION OF AMPLITUDE OF MECHANICAL DEFORMATION FOR MECHANICAL "FREQUENCY" = 5.

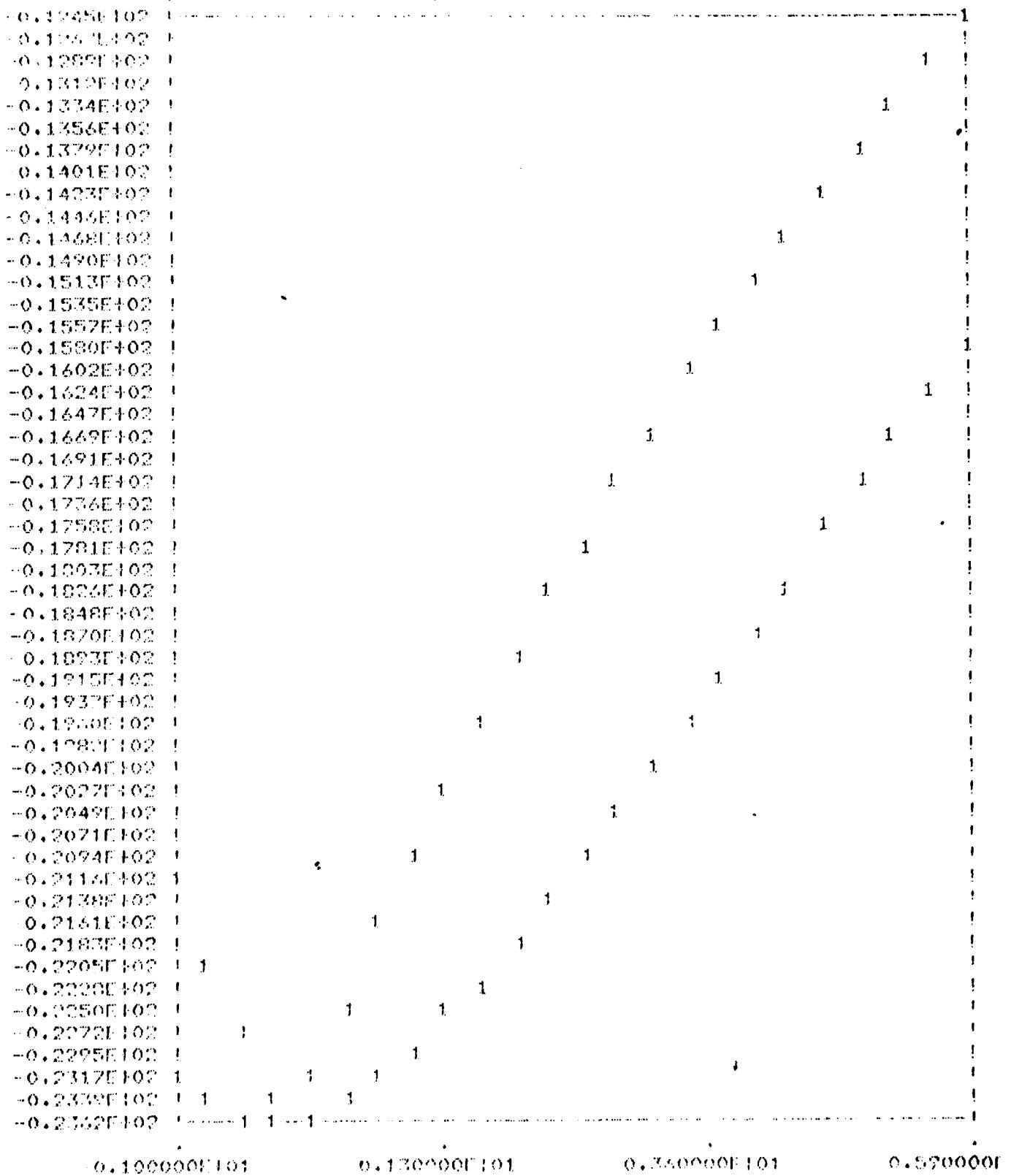


FIG. 9 - FIRST SIDE LOBE (DB), WITHOUT (UPPER) AND WITH (LOWER) PHASE CORRECTION AT SUB-ARRAY FEED, AS A FUNCTION OF AMPLITUDE OF MECHANICAL DEFORMATION FOR MECHANICAL "FREQUENCY" = 5.

-0.3145E-03	1
-0.1336E-01	* * * 1 * * *
-0.2441E-01	* 1
-0.3245E-01	1 1
-0.5250E-01	1 1
-0.6655E-01	1
-0.7859E-01	*
-0.9164E-01	1
-0.1047E+00	1
-0.1172E+00	1
-0.1308E+00	1
-0.1438E+00	1
-0.1562E+00	1
-0.1692E+00	1 1
-0.1830E+00	1
-0.1960E+00	1
-0.2091E+00	1
-0.2221E+00	1
-0.2351E+00	1
-0.2482E+00	1
-0.2612E+00	
-0.2743E+00	
-0.2873E+00	1
-0.3004E+00	1
-0.3134E+00	
-0.3265E+00	
-0.3395E+00	1 1
-0.3526E+00	
-0.3654E+00	
-0.3787E+00	
-0.3917E+00	1
-0.4047E+00	1
-0.4178E+00	
-0.4308E+00	
-0.4432E+00	1
-0.4569E+00	1
-0.4700E+00	
-0.4830E+00	
-0.4961E+00	
-0.5091E+00	1
-0.5222E+00	
-0.5352E+00	
-0.5483E+00	
-0.5613E+00	
-0.5744E+00	
-0.5874E+00	1
0.6004E+00	1
-0.6135E+00	
-0.6265E+00	
-0.6394E+00	
-0.6524E+00	1

0.100000E+01 0.150000E+01 0.200000E+01 0.590000E+01

FIG. 10 - MAIN BEAM (DB), WITHOUT (UPPER) AND WITH (LOWER) PHASE CORRECTION AT SUB-ARRAY FEED, AS A FUNCTION OF AMPLITUDE OF MECHANICAL DEFORMATION FOR MECHANICAL "FREQUENCY" = 10.

-0.1512E+02				1
-0.1522E+02				1
-0.1544E+02				1
-0.1562E+02				1
-0.1579E+02				1
-0.1594E+02				1
-0.1613E+02				1
-0.1630E+02				1
-0.1644E+02				1
-0.1663E+02				1
-0.1680E+02				1
-0.1697E+02				1
-0.1713E+02				1
-0.1730E+02				1
-0.1747E+02				1
-0.1764E+02				1
-0.1781E+02				1
-0.1797E+02				1
-0.1814E+02				1
-0.1831E+02				1
-0.1848E+02				1
-0.1864E+02				1
-0.1881E+02				1
-0.1898E+02				1
-0.1915E+02				1
-0.1932E+02				1
-0.1948E+02				1
-0.1965E+02				1
-0.1982E+02				1
-0.1999E+02				1
-0.2015E+02				1
-0.2032E+02				1
-0.2049E+02				1
-0.2066E+02				1
-0.2083E+02				1
-0.2099E+02				1
-0.2116E+02				1
-0.2133E+02				1
-0.2150E+02				1
-0.2166E+02				1
-0.2183E+02				1
-0.2200E+02	1	1		
-0.2217E+02				1
-0.2234E+02				1
-0.2250E+02	1			1
-0.2267E+02	1			1
-0.2284E+02				1
-0.2301E+02	1			1
-0.2317E+02				1
-0.2334E+02				1
-0.2351E+02				1
<hr/>				
-0.100000E+01	0.130000E+01	0.350000E+01	0.590000E+01	

FIG. 11 - FIRST SIDE LOBE (DB), WITHOUT (UPPER) AND WITH (LOWER) PHASE CORRECTION
AT SUB-ARRAY FEED, AS A FUNCTION OF AMPLITUDE OF MECHANICAL DEFORMATION
FOR MECHANICAL "FREQUENCY" = 10.

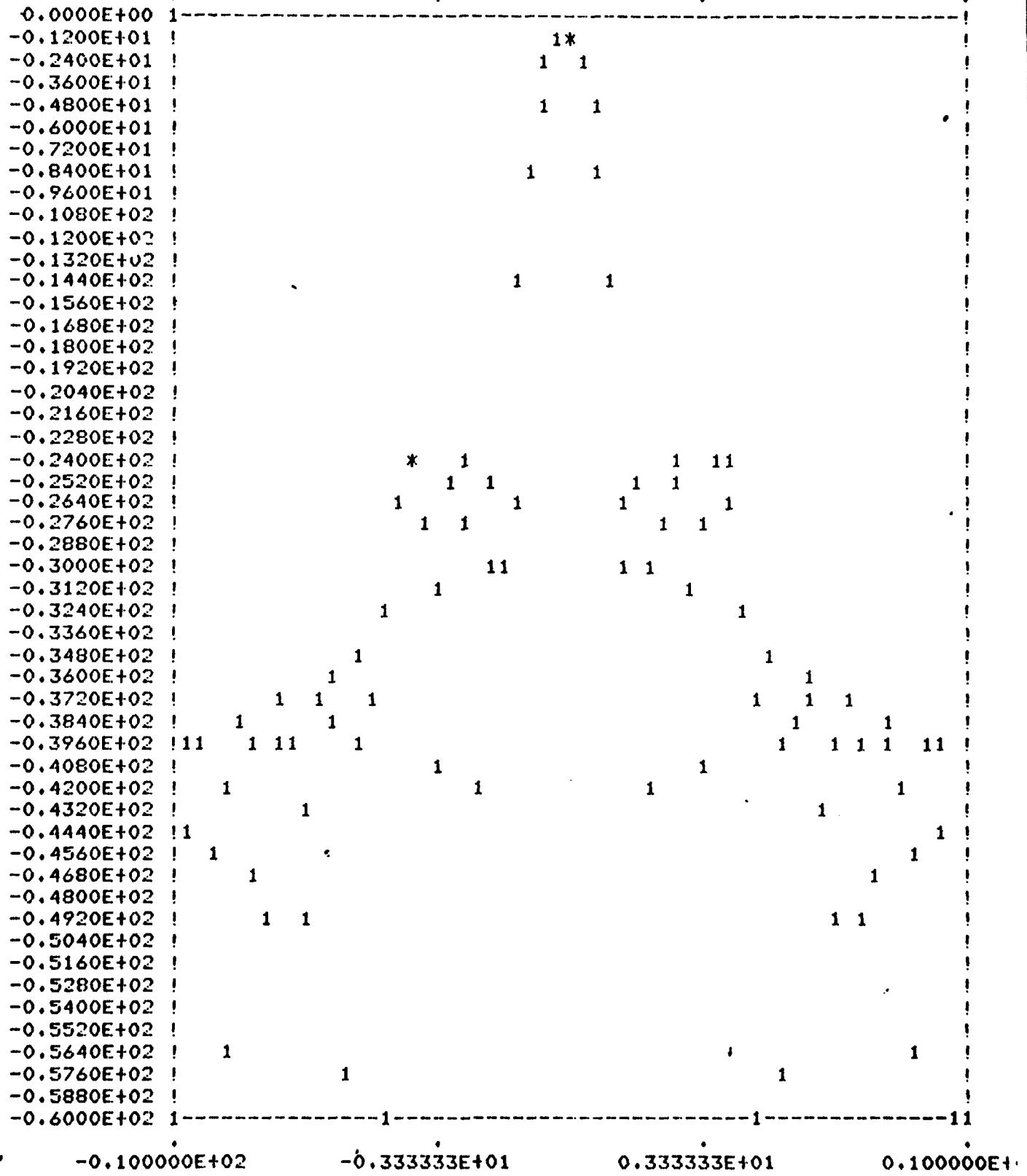


FIG. 12 - RADIATION PATTERN ALONG X-AXIS FOR TWO DIMENSIONAL LENS WITH NO MECHANICAL DEFORMATION.

0.0000E+00	1-----			
-0.1200E+01	!			1*
-0.2400E+01	!		1	1
-0.3600E+01	!			
-0.4800E+01	!		1	1
-0.6000E+01	!			
-0.7200E+01	!			
-0.8400E+01	!		1	1
-0.9600E+01	!			
-0.1080E+02	!			
-0.1200E+02	!		1	1
-0.1320E+02	!			
-0.1440E+02	!			
-0.1560E+02	!			
-0.1680E+02	!			
-0.1800E+02	!		1	1
-0.1920E+02	!			
-0.2040E+02	!			
-0.2160E+02	!			
-0.2280E+02	!			
-0.2400E+02	!		1	1
-0.2520E+02	!			
-0.2640E+02	!			
-0.2760E+02	!			
-0.2880E+02	!			
-0.3000E+02	!			
-0.3120E+02	!			
-0.3240E+02	!			
-0.3360E+02	!			
-0.3480E+02	!		1	1
-0.3600E+02	!			
-0.3720E+02	!			
-0.3840E+02	!			
-0.3960E+02	!			
-0.4080E+02	!		1	1
-0.4200E+02	!			
-0.4320E+02	!		1	1
-0.4440E+02	!			
-0.4560E+02	!		1	1
-0.4680E+02	!	11	1	
-0.4800E+02	!			1 *
-0.4920E+02	!	1	1	1 1 1 1
-0.5040E+02	!			
-0.5160E+02	!		1	1
-0.5280E+02	!			
-0.5400E+02	!	1		1
-0.5520E+02	!		1	
-0.5640E+02	!	1	1	1 1
-0.5760E+02	!			
-0.5880E+02	!			
-0.6000E+02	1*11*1*1*1*1-----*---11-----11---11---11*1*1*1*1*11			
-0.100000E+02		-0.333333E+01	0.333333E+01	0.100000E+

FIG. 13 - RADIATION PATTERN AT 45° X-AXIS FOR TWO DIMENSIONAL LENS WITH NO MECHANICAL DEFORMATION.

FIG. 14 - RADIATION PATTERN ALONG Y-AXIS FOR TWO DIMENSIONAL LENS WITH NO MECHANICAL DEFORMATION.

```

0.0000E+00 1-----1*
-0.1200E+01 1
-0.2400E+01 1 1
-0.3600E+01 1 1
-0.4800E+01 1 1
-0.6000E+01 1 1
-0.7200E+01 1
-0.8400E+01 1 1
-0.9600E+01 1
-0.1080E+02 1
-0.1200E+02 1 1
-0.1320E+02 1 1
-0.1440E+02 1
-0.1560E+02 1
-0.1680E+02 1
-0.1800E+02 1
-0.1920E+02 1 1
-0.2040E+02 1
-0.2160E+02 1
-0.2280E+02 1
-0.2400E+02 1
-0.2520E+02 1
-0.2640E+02 1
-0.2760E+02 1
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-0.3120E+02 1 1
-0.3240E+02 1
-0.3360E+02 1
-0.3480E+02 1
-0.3600E+02 1
-0.3720E+02 1
-0.3840E+02 1
-0.3960E+02 1 1
-0.4080E+02 1
-0.4200E+02 1
-0.4320E+02 1
-0.4440E+02 1 1
-0.4560E+02 1
-0.4680E+02 1
-0.4800E+02 1 1
-0.4920E+02 1 1
-0.5040E+02 1 1
-0.5160E+02 1
-0.5280E+02 1
-0.5400E+02 1 1
-0.5520E+02 1 1
-0.5640E+02 1
-0.5760E+02 1
-0.5880E+02 1
-0.6000E+02 1*11*1*111*111-*11-1--1-----1-1-11-11-*1*-*1*1*1*11

-0.100000E+02 -0.333333E+01 0.333333E+01 0.100000E+02

```

FIG. 15 - RADIATION PATTERN AT 120° TO X-AXIS FOR TWO DIMENSIONAL LENS WITH NO MECHANICAL DEFORMATION.

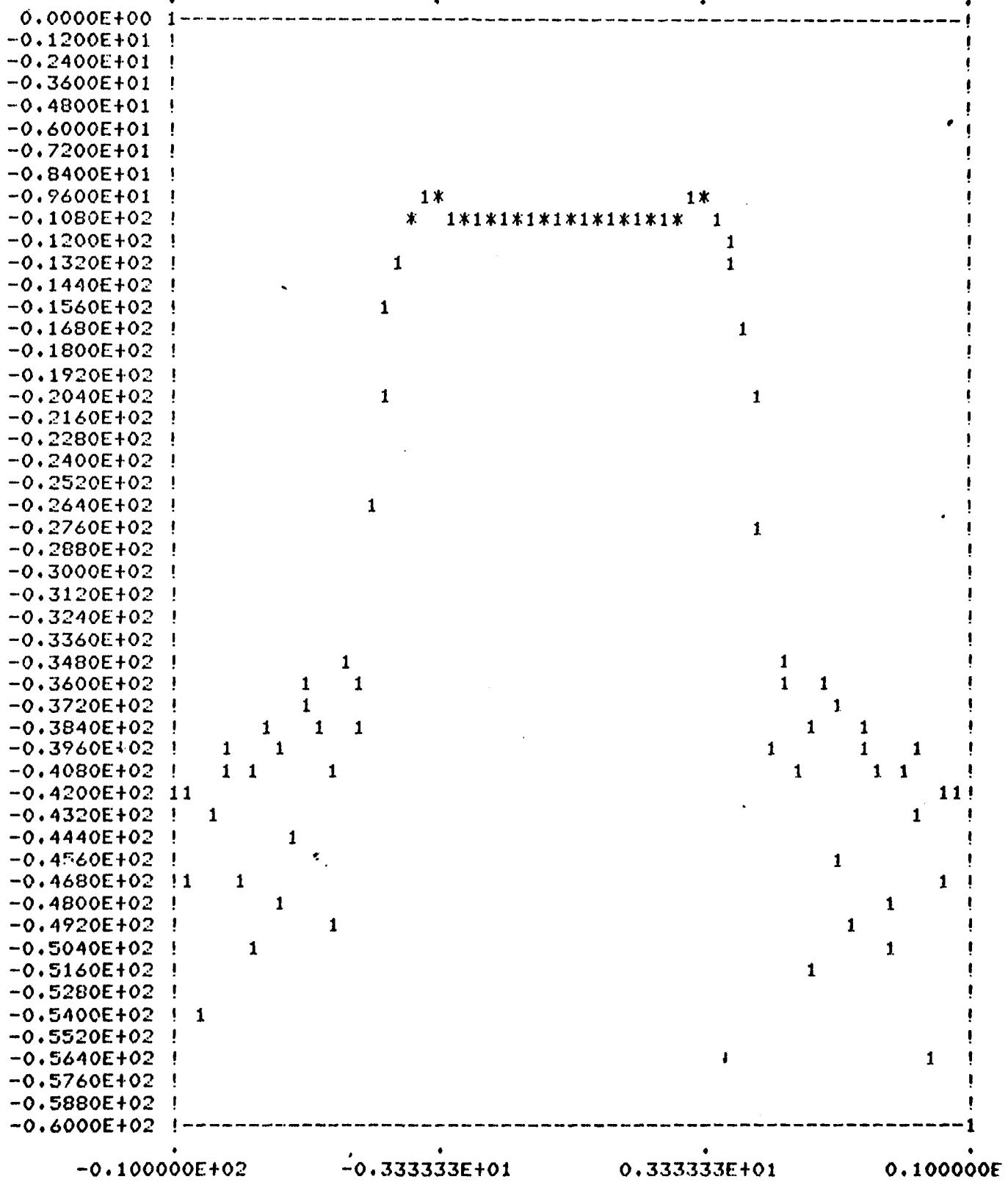


FIG. 16 - ELEMENT PATTERN ALONG X-AXIS FOR TWO DIMENSIONAL LENS WITH NO MECHANICAL DEFORMATION.

0.0000E+00	1			
-0.1200E+01				
-0.2400E+01				
-0.3600E+01				
-0.4800E+01				
-0.6000E+01				
-0.7200E+01				
-0.8400E+01		11		*
-0.9600E+01		1 11		*1 11
-0.1080E+02		1 * 1*1*1*1*1*1*1*1*1		
-0.1200E+02		1 1*1*		1
-0.1320E+02				1
-0.1440E+02		1		
-0.1560E+02				1
-0.1680E+02		1		
-0.1800E+02				1
-0.1920E+02		1		
-0.2040E+02				1
-0.2160E+02				
-0.2280E+02		1		
-0.2400E+02				
-0.2520E+02				1
-0.2640E+02		1		
-0.2760E+02				
-0.2880E+02				
-0.3000E+02		1		
-0.3120E+02				
-0.3240E+02		1		1
-0.3360E+02				
-0.3480E+02		1		
-0.3600E+02				
-0.3720E+02		1		
-0.3840E+02				
-0.3960E+02		1		
-0.4080E+02				
-0.4200E+02				*
-0.4320E+02				1 1
-0.4440E+02		1		
-0.4560E+02				11
-0.4680E+02				1 1 1
-0.4800E+02				
-0.4920E+02	11			1
-0.5040E+02				1
-0.5160E+02	11 1			
-0.5280E+02		1		
-0.5400E+02				
-0.5520E+02	11			1
-0.5640E+02				1 1 1
-0.5760E+02				
-0.5880E+02				
-0.6000E+02	1---1---			1
	-0.100000E+02		-0.333333E+01	0.333333E+01
				0.100000E+

FIG. 17 - ELEMENT PATTERN AT 45° TO X-AXIS FOR TWO DIMENSIONAL LENS WITH NO MECHANICAL DEFORMATION.

```

0.0000E+00 1
-0.1200E+01
-0.2400E+01
-0.3600E+01
-0.4800E+01
-0.6000E+01
-0.7200E+01
-0.8400E+01
-0.9600E+01
-0.1080E+02
-0.1200E+02
-0.1320E+02
-0.1440E+02
-0.1560E+02
-0.1680E+02
-0.1800E+02
-0.1920E+02
-0.2040E+02
-0.2160E+02
-0.2280E+02
-0.2400E+02
-0.2520E+02
-0.2640E+02
-0.2760E+02
-0.2880E+02
-0.3000E+02
-0.3120E+02
-0.3240E+02
-0.3360E+02
-0.3480E+02
-0.3600E+02
-0.3720E+02
-0.3840E+02
-0.3960E+02
-0.4080E+02
-0.4200E+02
-0.4320E+02
-0.4440E+02
-0.4560E+02
-0.4680E+02
-0.4800E+02
-0.4920E+02
-0.5040E+02
-0.5160E+02
-0.5280E+02
-0.5400E+02
-0.5520E+02
-0.5640E+02
-0.5760E+02
-0.5880E+02
-0.6000E+02
-0.100000E+02
-0.333333E+01
0.333333E+01
0.100000E+

```

FIG. 18 - ELEMENT PATTERN ALONG Y-AXIS FOR TWO DIMENSIONAL LENS WITH NO MECHANICAL DEFORMATION.

0.0000E+00	1-----
-0.1200E+01	1
-0.2400E+01	1
-0.3600E+01	1
-0.4800E+01	1
-0.6000E+01	1
-0.7200E+01	1
-0.8400E+01	1
-0.9600E+01	1*1
-0.1080E+02	1 *1 11*1*1*1*1*1*1*1*1*1
-0.1200E+02	1 *11 1
-0.1320E+02	1
-0.1440E+02	1
-0.1560E+02	1
-0.1680E+02	1
-0.1800E+02	1
-0.1920E+02	1
-0.2040E+02	1
-0.2160E+02	1
-0.2280E+02	1
-0.2400E+02	1
-0.2520E+02	1
-0.2640E+02	1
-0.2760E+02	1
-0.2880E+02	1
-0.3000E+02	1
-0.3120E+02	1
-0.3240E+02	11
-0.3360E+02	11 1
-0.3480E+02	1 1
-0.3600E+02	1*111 *
-0.3720E+02	1
-0.3840E+02	1
-0.3960E+02	1 1
-0.4080E+02	1
-0.4200E+02	1
-0.4320E+02	1
-0.4440E+02	1
-0.4560E+02	1
-0.4680E+02	1 1
-0.4800E+02	1
-0.4920E+02	1
-0.5040E+02	1
-0.5160E+02	1
-0.5280E+02	1
-0.5400E+02	1
-0.5520E+02	1
-0.5640E+02	1
-0.5760E+02	1
-0.5880E+02	1
-0.6000E+02	1-----
-0.100000E+02 -0.333333E+01 0.333333E+01 0.100000E+02	

FIG. 19 - ELEMENT PATTERN AT 120° TO X-AXIS FOR TWO DIMENSIONAL LENS WITH NO MECHANICAL DEFORMATION.

FIG. 20 - RADIATION PATTERN ALONG X-AXIS FOR TWO DIMENSIONAL LENS WITH AMPLITUDE OF MECHANICAL DEFORMATION = 1 AND MECHANICAL "FREQUENCY" = 2.

0.0000E+00	1			
-0.1200E+01				
-0.2400E+01		1*		
-0.3600E+01		1	1	
-0.4800E+01				
-0.6000E+01		1	1	
-0.7200E+01				
-0.8400E+01				
-0.9600E+01		1	1	
-0.1080E+02				
-0.1200E+02		11*	1*1	
-0.1320E+02		1		1
-0.1440E+02				
-0.1560E+02		1		1
-0.1680E+02				
-0.1800E+02		1		1
-0.1920E+02				
-0.2040E+02				
-0.2160E+02		1		1
-0.2280E+02				
-0.2400E+02				
-0.2520E+02				
-0.2640E+02		1		1
-0.2760E+02		*		11
-0.2880E+02		1		1
-0.3000E+02				
-0.3120E+02		1		1
-0.3240E+02				
-0.3360E+02				
-0.3480E+02		1		1
-0.3600E+02				
-0.3720E+02				
-0.3840E+02		1		1
-0.3960E+02				
-0.4080E+02				
-0.4200E+02				
-0.4320E+02		1		1
-0.4440E+02				
-0.4560E+02				
-0.4680E+02		1		1
-0.4800E+02		1*1		11*
-0.4920E+02				
-0.5040E+02		1		1
-0.5160E+02				
-0.5280E+02		1		1
-0.5400E+02		1	1	1
-0.5520E+02		1		1
-0.5640E+02				
-0.5760E+02		1		1
-0.5880E+02		11		11
-0.6000E+02	1*11*1*1			11*1*1*11

-0.100000E+02 -0.333333E+01 0.333333E+01 0.100000E+

FIG. 21 - RADIATION PATTERN AT 45° TO X-AXIS FOR TWO DIMENSIONAL LENS WITH AMPLITUDE OF MECHANICAL DEFORMATION = 1 AND MECHANICAL "FREQUENCY" = 2.

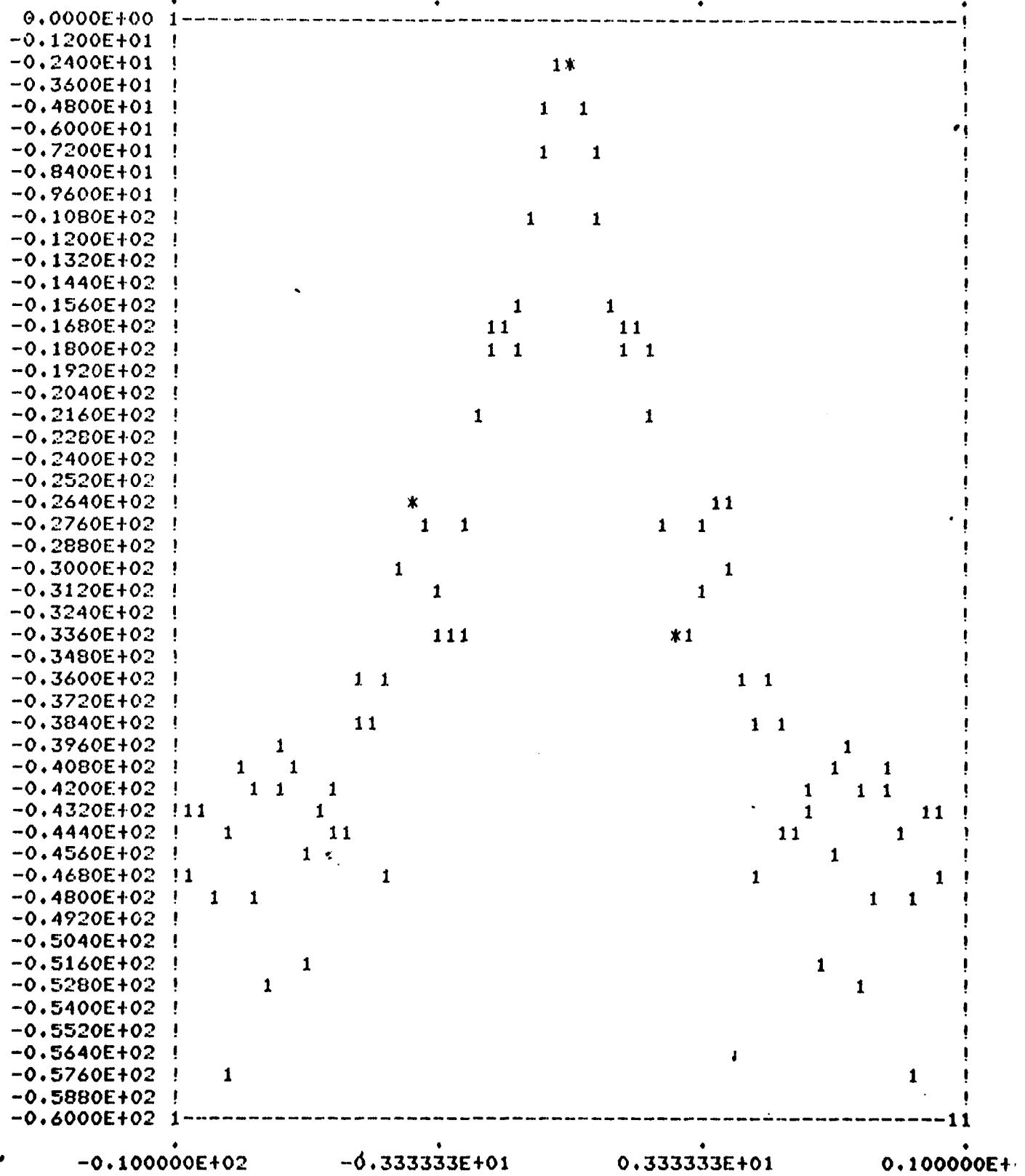


FIG. 22 - RADIATION PATTERN ALONG Y-AXIS FOR TWO DIMENSIONAL LENS WITH AMPLITUDE OF MECHANICAL DEFORMATION = 1 AND MECHANICAL "FREQUENCY" = 2.

0.0000E+00	1			
-0.1200E+01				
-0.2400E+01		1*		
-0.3600E+01			1	1
-0.4800E+01				
-0.6000E+01			1	1
-0.7200E+01				
-0.8400E+01				
-0.9600E+01			1	1
-0.1080E+02				
-0.1200E+02		1		
-0.1320E+02		111		*1
-0.1440E+02				
-0.1560E+02		1		1
-0.1680E+02				
-0.1800E+02		1		1
-0.1920E+02				
-0.2040E+02				
-0.2160E+02		1		1
-0.2280E+02				
-0.2400E+02				
-0.2520E+02				
-0.2640E+02				
-0.2760E+02		* 1		111
-0.2880E+02		1		1
-0.3000E+02				
-0.3120E+02		1		1
-0.3240E+02				
-0.3360E+02				
-0.3480E+02				
-0.3600E+02		1		1
-0.3720E+02				
-0.3840E+02				
-0.3960E+02				
-0.4080E+02				
-0.4200E+02		1		1
-0.4320E+02		*		11
-0.4440E+02		1		1
-0.4560E+02				
-0.4680E+02		1		1
-0.4800E+02		1		1
-0.4920E+02		1		1
-0.5040E+02		*		11
-0.5160E+02		1		1
-0.5280E+02				
-0.5400E+02				
-0.5520E+02				
-0.5640E+02		1		1
-0.5760E+02				
-0.5880E+02				
-0.6000E+02	1*11*1*1*1*	-----	-----	1*1*1*1*1*11
-0.100000E+02		-0.333333E+01	0.333333E+01	0.100000E+

FIG. 23 - RADIATION PATTERN AT 120° TO X-AXIS FOR TWO DIMENSIONAL LENS WITH AMPLITUDE OF MECHANICAL DEFORMATION = 1 AND MECHANICAL "FREQUENCY" = 2.

```

0.0000E+00 1-----1
-0.1200E+01 !                               1*
-0.2400E+01 !                               1   1
-0.3600E+01 !
-0.4800E+01 !
-0.6000E+01 !
-0.7200E+01 !
-0.8400E+01 !
-0.9600E+01 !                               1   1
-0.1080E+02 !
-0.1200E+02 !
-0.1320E+02 !
-0.1440E+02 !
-0.1560E+02 !                               1   1
-0.1680E+02 !
-0.1800E+02 !
-0.1920E+02 !
-0.2040E+02 !
-0.2160E+02 !
-0.2280E+02 !
-0.2400E+02 !
-0.2520E+02 !                               1   * 1   1   1   1
-0.2640E+02 !
-0.2760E+02 !                               1   1   1   1   1   1   1
-0.2880E+02 !
-0.3000E+02 !     11                      11   1   1   11
-0.3120E+02 !     1 1                      1           1       1 1
-0.3240E+02 !             1
-0.3360E+02 !
-0.3480E+02 !     1 1                      11   11   1   1
-0.3600E+02 !
-0.3720E+02 !             1
-0.3840E+02 !     1 1 1
-0.3960E+02 !     1 1 1   1
-0.4080E+02 !
-0.4200E+02 !
-0.4320E+02 !
-0.4440E+02 !
-0.4560E+02 11
-0.4680E+02 !     1
-0.4800E+02 !     1
-0.4920E+02 !     1 1
-0.5040E+02 !
-0.5160E+02 !
-0.5280E+02 !
-0.5400E+02 !
-0.5520E+02 !
-0.5640E+02 !
-0.5760E+02 !
-0.5880E+02 !
-0.6000E+02 !-----1

```

FIG. 24 - RADIATION PATTERN ALONG X-AXIS FOR TWO DIMENSIONAL LENS WITH AMPLITUDE OF MECHANICAL DEFORMATION = 1 AND MECHANICAL "FREQUENCY" = 8.

0:0000E+00	1-----					
-0.1200E+01						1*
-0.2400E+01					1	1
-0.3600E+01					1	1
-0.4800E+01					1	1
-0.6000E+01					1	1
-0.7200E+01					1	1
-0.8400E+01					1	1
-0.9600E+01					1	1
-0.1080E+02					1	1
-0.1200E+02					1	1
-0.1320E+02					1	1
-0.1440E+02					1	1
-0.1560E+02					1	1
-0.1680E+02		11				*
-0.1800E+02		1 1				1 1
-0.1920E+02		1 1		1	1	1 1
-0.2040E+02						1
-0.2160E+02			1			1
-0.2280E+02		1				1
-0.2400E+02						1
-0.2520E+02			1			1
-0.2640E+02		1		1	1	1
-0.2760E+02						1
-0.2880E+02			1			1
-0.3000E+02						1
-0.3120E+02		1				1
-0.3240E+02						1
-0.3360E+02						1
-0.3480E+02			1	1	1	1
-0.3600E+02						1
-0.3720E+02		1				1
-0.3840E+02						1
-0.3960E+02						1
-0.4080E+02						1
-0.4200E+02			1	1	1	1
-0.4320E+02						1
-0.4440E+02		1		1	1	1
-0.4560E+02				1	1	
-0.4680E+02						1
-0.4800E+02			1			1
-0.4920E+02			1 1 1		1	*
-0.5040E+02						1
-0.5160E+02				1		1
-0.5280E+02		1				1
-0.5400E+02						1
-0.5520E+02						1
-0.5640E+02				1		1
-0.5760E+02		1				1
-0.5880E+02						1
-0.6000E+02	1*11*	-----	11-----	-----	11-----	1*1*11
-0.100000E+02		-0.333333E+01		0.333333E+01		0.100000E+

FIG. 25 - RADIATION PATTERN AT 45° TO X-AXIS FOR TWO DIMENSIONAL LENS WITH AMPLITUDE OF MECHANICAL DEFORMATION = 1 AND MECHANICAL "FREQUENCY" = 2.

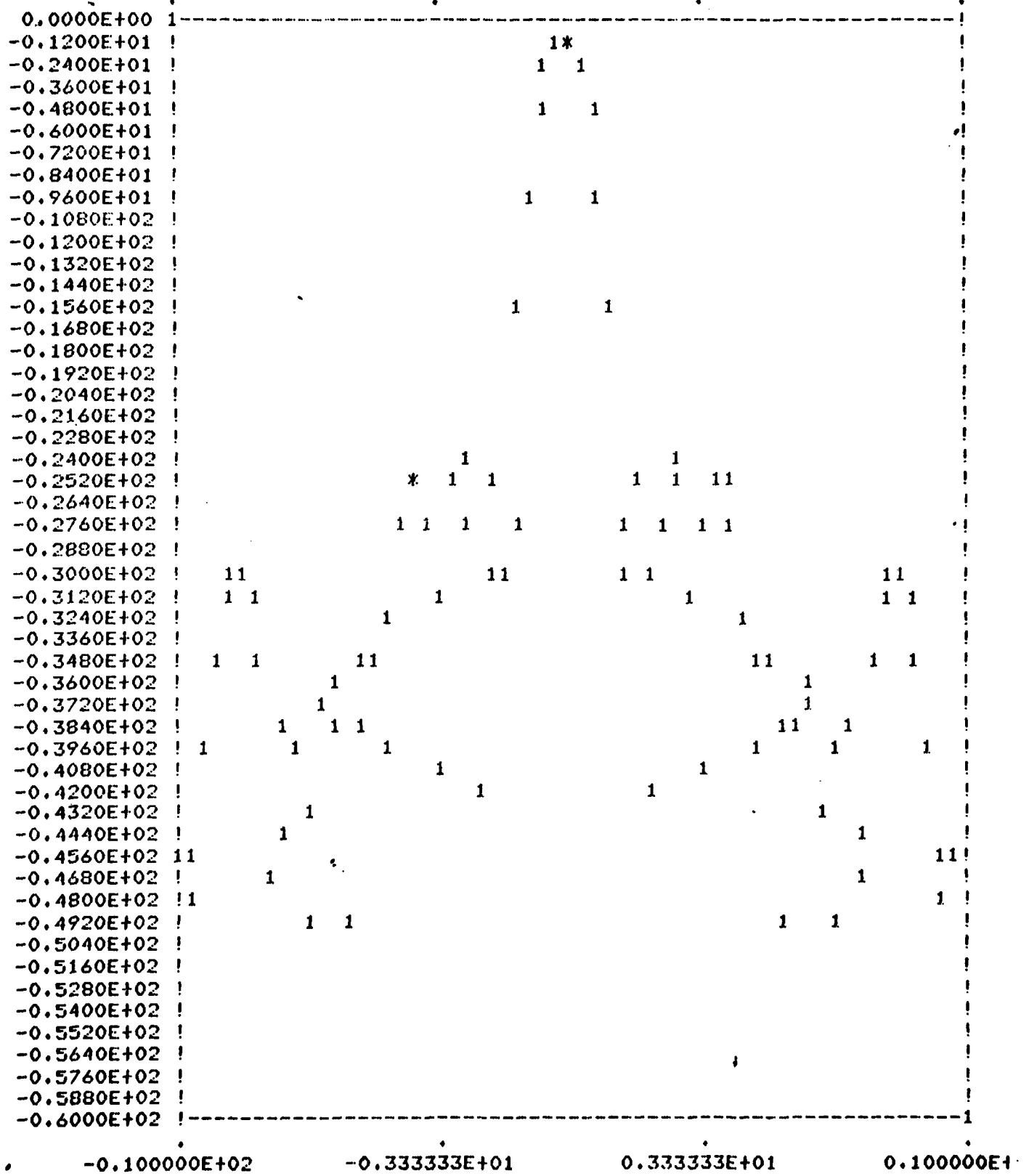


FIG. 26 - RADIATION PATTERN ALONG Y-AXIS FOR TWO DIMENSIONAL LENS WITH AMPLITUDE OF MECHANICAL DEFORMATION = 1 AND MECHANICAL "FREQUENCY" = 2.

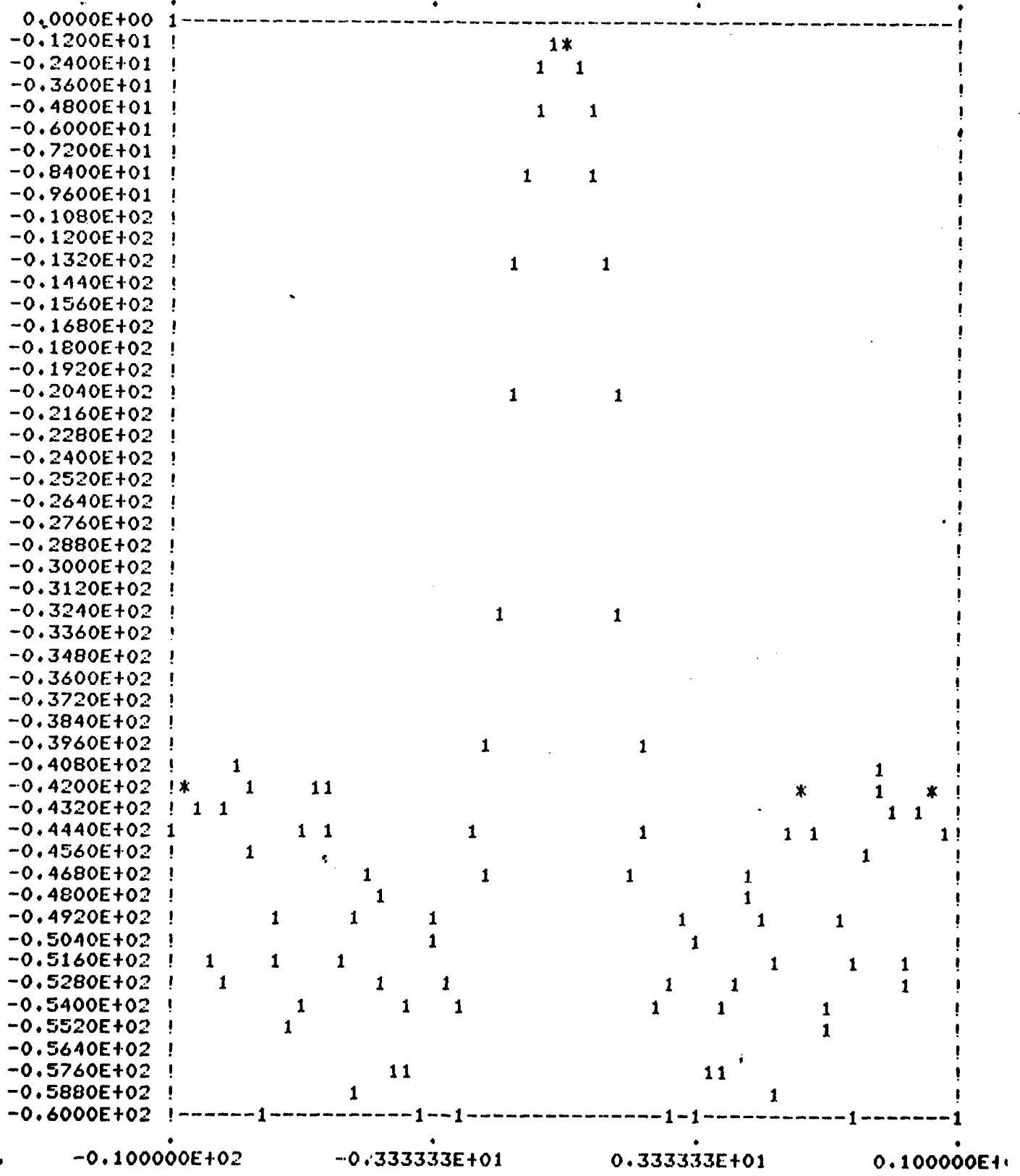


FIG. 27 - RADIATION PATTERN AT 120° TO X-AXIS FOR TWO DIMENSIONAL LENS WITH AMPLITUDE OF MECHANICAL DEFORMATION = 1 AND MECHANICAL "FREQUENCY" = 2.